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Study of rocking motion of rigid body with slide contact †

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Abstract

The purpose of this study is to formulate the motion of a rigid body with unilateral contact problems by applying techniques of multibody dynamics and to analyze the issue of rocking condition of rigid bodies with slide contact. In To investigate rocking motion with slide contact, we formulate for dynamics of a simple rigid body system with a unilateral contact model. Judgment for the occurrence of contact between a rigid body and a base is applied. The planar motion of a rigid body system having a simple shape and both with and without slide cases is assumed. Using constraint conditions for the contact as algebraic equations, the rocking motion of the rigid body, including slide and frictional force, is analyzed. The differential algebraic equation is solved by the augmented method with Lagrange multipliers, using generalized coordinates and independent variables that describe the contact points. The influence of the frequency and amplitude of disturbance given to the base is discussed.

Keywords: Rocking motion; Slide contact; Unilateral contact; Overturning

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1. Introduction

The mechanism and the condition of the rocking motion of a rigid body are focused on in this paper. The nature of rocking motions is not only influenced by factors such as the contact geometry and physical interaction between rigid bodies and the bases, but also by physical characteristics of the applied disturbance. There are many engineering problems to be solved, applying multibody dynamics techniques, considering contact geometry, impact, slide and so on.

For example, the safety and stability issues of railroad vehicles experiencing a great earthquake are typical examples of such a problem. To secure their safety and stability, it is also important to make arguments based on theoretical and analytical approaches. The rocking motion of the tanks has also been studied [1]. For that case, a dynamical system of the rocking motion of the tanks was formulated by employing the effective mass of the liquid for rocking motion and the contribution of the rotational inertia forces. A machine tool, a set of computer equipment, a piece of furniture, and a simple architectural element all can be modeled as a rigid body rocking on a rigid foundation. In previous studies, overturning phenomena of the rocking rigid bodies were discussed, in order to limit the response severity and to identify the factors needed to avoid overturning [2, 3]. However, many of these studies were applied with theoretical analysis, that is, a linearization of the nonlinear equation of motion, which ignores the contact geometry and the slide contact between the rigid body and the base. So, to precisely analyze rocking problems, a slide must be considered at the contact points. In addition, the shape of the contact surfaces often plays a significant role in the rocking motion.

In this study, modeling and formulation for the rocking motion of the rigid body on the base are car-

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ried out, considering contact geometry, impact and slide. The effect of the slide on the overturning is clarified with numerical results.

2. Modeling and formulation

2.1 Modeling

In this chapter, a numerical model of a rocking rigid body with slide contact between the rigid body and the base is described. The rocking motion of a rigid body with and without slide contact is discussed. A horizontal excitation is added to the base. The contact geometry between the rigid body and the base can be given. Here, to simplify the problem, it is assumed that the contact surface between the rigid body and the base is a line and arc as shown in Fig. 1. Slide on the contact point is also taken into consideration. The right and left contact points are denoted as A and B, respectively. The base and the rigid body are assumed to contact without failure at either contact point. For simplification, the contact between the rigid body and the base is assumed to be a point contact. In this case, the motion of the rigid body has one or two d.o.f during rocking. Without slide, it has only rolling motion. On the other hand, with slide, it has both rolling and translation motion. The position of the contact point without slide is determined according to the rotation angle of the rigid body and the radius of the base. The transfer of the contact point with slide along the line on the rigid body and the arc on the base is determined according to the independent variables that describe the contact point. A frictional force acts on the contact point in the case with slide.

2.2 Formulation

The equations of motion for rocking of the rigid body are derived, considering the contact geometry. Formulation is completed for rocking motion on each base with and without slide. The orthogonal coordinate $\mathrm{O-XY}$ is the fixed frame. On the other hand, $\mathrm{G-xy}$ is fixed on the center of mass of the rigid body and moves and rotates with the rigid body. θ is the angular displacement of the rigid body. The contact point P_0 is regarded as the basis when the angular displacement θ is zero.

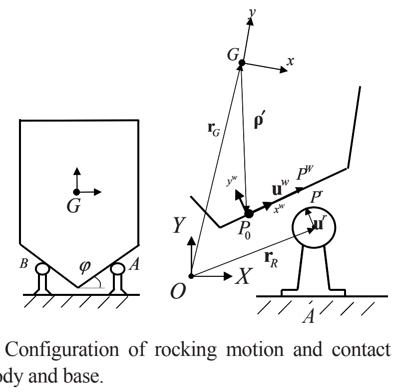


Fig. 1. Configuration of rocking motion and contact between rigid body and base.

2.2.1 Without slide contact

This section addresses the case without slide. The equation of motion for rocking of the rigid body in the fixed frame $\mathrm{O-XY}$ is written as,

$$\mathbf{M}\ddot{\mathbf{r}}_G + \overline{\mathbf{F}} = \mathbf{F}, \tag{1}$$

$$I\ddot{\theta} + \overline{N} = N \tag{2}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{r}_G$ is the position vector to the mass center, $\mathbf{F}$ is an external force, and $\overline{\mathbf{F}}$ is an unknown constraint force acting on the rigid body. I is inertia moment, N is external torque and $\overline{N}$ is unknown constraint torque. The constraint for the unilateral contact between the rigid body and the base is derived. In Fig. 1, P^w denotes the contact point between the rigid body and the base and is on the rigid body. The position vector $\mathbf{r}^w$ is defined as,

$$\mathbf{r}^w = \mathbf{r}_G + \mathbf{A}\boldsymbol{\rho}' + \mathbf{u}^w \tag{3}$$

where position vector $\boldsymbol{\rho}'$ is for the basis P_0 on the rigid body written in the body-fixed coordinate system $\mathrm{G-xy}$. $\mathbf{A}$ is transformation matrix from $\mathrm{G-xy}$ to $\mathrm{O-XY}$. $\mathbf{u}^w$ is position vector for contact point on the rigid body. When the rigid body rotates θ, the contact point moves over $|a - r\phi|$, so the constraint for without slide is,

$$|\mathbf{u}^w| = |a - r\phi| \tag{4}$$

where a is the distance between the basis and the contact point on the rigid body. r is a radius of the base. ϕ is the initial angular displacement of the rigid body. φ is angular displacement of the rigid body's shape. On the other hand, P^r denotes the contact point between the rigid body and the base, which is on the base. The position vector $\mathbf{r}^r$ is defined as,

$$\mathbf{r}^r = \mathbf{r}_R + \mathbf{u}^r \tag{5}$$

where $\mathbf{u}^r$ is position vector for the contact point on the base.

The constraint equation for without slide contact between the rigid body and the base is described by,

$$\mathbf{C}(\mathbf{q},t)=\left[\mathbf{r}^{w}-\mathbf{r}^{r}\right]=0 \tag{6}$$

Finally, the differential algebraic equations can be written as,

$$\begin{bmatrix}\mathbf{M} & \mathbf{C}_{\mathbf{q}}^{T}\\ \mathbf{C}_{\mathbf{q}} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\ddot{\mathbf{q}}\\ \boldsymbol{\lambda}\end{bmatrix}=\begin{bmatrix}\mathbf{F}\\ \boldsymbol{\gamma}\end{bmatrix} \tag{7}$$

Where $\mathbf{q}$ is the vector of generalized coordinate, $\mathbf{C_q}$ is the constraint Jacobian matrix, $\boldsymbol{\lambda}$ is the vector of Lagrange multipliers. Then $\mathbf{C_q}$ can be obtained as,

$$\mathbf{C}_{\mathbf{q}}=\frac{\partial\mathbf{C}(\mathbf{q},t)}{\partial\mathbf{q}} \tag{8}$$

2.2.2 With slide contact

The constraint equation with the generalized coordinates[4] and the independent variables[5, 6] that describe the contact points is derived. The frictional force f_T acts on the contact point and the external force is represented by,

$$\mathbf{F}=\begin{bmatrix} f_T\cos(\varphi+\theta)\\ -Mg+f_T\sin(\varphi+\theta)\end{bmatrix}, \tag{9}$$

$$N=Rf_T \tag{10}$$

where M is mass and R is effort arm of the rigid body. In Fig. 1, P^w denotes the contact point between the rigid body and the base and is on the rigid body. The position vector $\hat{\mathbf{r}}^w$ is defined as,

$$\hat{\mathbf{r}}^{w}=\mathbf{r}_{G}+\mathbf{A}\boldsymbol{\rho}'+\hat{\mathbf{u}}^{w} \tag{11}$$

The position vector $\hat{\mathbf{u}}^w$ is for the contact point on the rigid body, and is written in the body fixed coordinate system $\mathrm{P^w-x^wy^w}$. s_1 is the independent variable that describes the contact points on the rigid body. $\hat{\mathbf{u}}^w$ is the position vector to the contact point as follows:

$$\hat{\mathbf{u}}^{w}=\begin{bmatrix}s_1\cos(\varphi+\theta)\\ s_1\sin(\varphi+\theta)\end{bmatrix} \tag{12}$$

On the other hand, P^r denotes the contact point between the rigid body and the base and is on the base. The position vector $\hat{\mathbf{r}}^r$ is defined as,

$$\hat{\mathbf{r}}^{r}=\mathbf{r}_{R}+\hat{\mathbf{u}}^{r} \tag{13}$$

where $\hat{\mathbf{u}}^r$ is the position vector to the contact point on the base. s_2 is the independent variable which describes contact points on the base. $\hat{\mathbf{u}}^r$ is written as,

$$\hat{\mathbf{u}}^{r}=\begin{bmatrix}r\cos(s_2)\\ r\sin(s_2)\end{bmatrix} \tag{14}$$

The constraints for the contact attitude along the tangential line in common are also described. The unit vectors in the tangential direction on the rigid body and in the normal direction on the base, $\mathbf{t}^w$ and $\mathbf{n}^r$, respectively, can be obtained as,

$$\mathbf{t}^{w}=\begin{bmatrix}\cos(\varphi+\theta)\\ \sin(\varphi+\theta)\end{bmatrix}, \tag{15}$$

$$\mathbf{n}^{r}=\begin{bmatrix}-\cos(s_2)\\ -\sin(s_2)\end{bmatrix} \tag{16}$$

The constraints for the case with slide contact between the rigid body and the base are obtained as,

$$\mathbf{C}(\mathbf{q},\mathbf{s},t)=\begin{bmatrix}\mathbf{r}^{w}-\mathbf{r}^{r}\\ \mathbf{t}^{wT}\mathbf{n}^{r}\end{bmatrix}=0 \tag{17}$$

where, $\mathbf{s}$ is the vector of independent variables.
Finally, the differential algebraic equations can be written as,

$$\begin{bmatrix}\mathbf{M} & \mathbf{0} & \mathbf{C}_{\mathbf{q}}^{T}\\ \mathbf{0} & \mathbf{0} & \mathbf{C}_{\mathbf{s}}^{T}\\ \mathbf{C}_{\mathbf{q}} & \mathbf{C}_{\mathbf{s}} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\ddot{\mathbf{q}}\\ \ddot{\mathbf{s}}\\ \boldsymbol{\lambda}\end{bmatrix}=\begin{bmatrix}\mathbf{F}\\ \mathbf{0}\\ \boldsymbol{\gamma}\end{bmatrix} \tag{18}$$

where $\mathbf{C_s}$ is the constraint Jacobian matrix. Then, $\mathbf{C_q}$ and $\mathbf{C_s}$ are given by,

$$\mathbf{C}_{\mathbf{q}}=\frac{\partial\mathbf{C}(\mathbf{q},\mathbf{s},t)}{\partial\mathbf{q}}, \tag{19}$$

$$\mathbf{C}_{\mathbf{s}}=\frac{\partial\mathbf{C}(\mathbf{q},\mathbf{s},t)}{\partial\mathbf{s}} \tag{20}$$

The frictional force in the case with slide is written as,

$$f_T=\mu\{-\lambda_X\sin(\varphi+\theta)+\lambda_Y\cos(\varphi+\theta)\} \tag{21}$$

where λ_X and λ_Y are the unknown constraint forces from Equation (18) and act on the contact point. Then direction of the frictional force is judged according to the slide velocity at the contact point. The slide velocity is relative and is expressed as the difference between the translation velocity and the rotation velocity at the contact point.

3. Judgment for slide and contact

3.1 Judgment for slide generation and stop

To judge the slide generation, the difference between the translation velocity and the rotation velocity of the rigid body must be monitored in the calculation, by using the following conditions [7, 8].

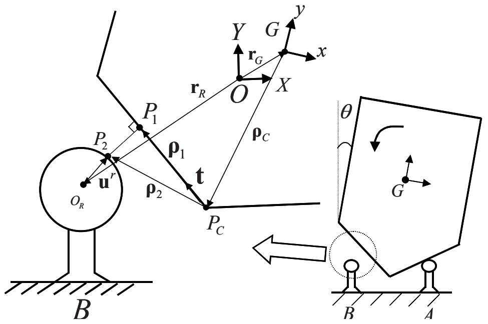


Fig. 2. Definition of vector.

Slide generation:

$$\lambda_t > \mu_0 \lambda_n \tag{22}$$

where λ_t and λ_n are contact forces in tangential and normal direction. μ_0 is the coefficient of friction.
End of slide:

$$\dot{s}_1 - r\dot{\theta} < \varepsilon \tag{23}$$

where ε is a small parameter $\varepsilon << 1$.

3.2 Judgment for contact during rocking

To judge for the contact during rocking motion, the distance between the contacting points on the base and the rigid body must be observed. The conditions at the contact point B are now described. The model for the contact during the rocking motion is shown in Fig. 2. $\boldsymbol{\rho}_1$ is the position vector from P_C to P_1 on the rigid body, $\boldsymbol{\rho}_2$ is the position vector from P_C to P_2 on the base, and $\mathbf{t}$ is the tangential unit vector on the contact point of the rigid body.

P_1 and P_2 are the contact points on the rigid body and the base, respectively. Applying the scalar product, the distance d between P_C to P_1 can be obtained as,

$$d = \mathbf{t} \cdot \boldsymbol{\rho}_2 \tag{24}$$

The position vector $\boldsymbol{\rho}_1$ is given by,

$$\boldsymbol{\rho}_1 = d \cdot \mathbf{t} \tag{25}$$

Then the position vector $\mathbf{r}_{P1}$ from origin O to point P_1 is written as,

$$\mathbf{r}_{P1} = \mathbf{r}_G + \boldsymbol{\rho}_C + \boldsymbol{\rho}_1 \tag{26}$$

The condition for the occurrence of contact between the rigid body and the base is calculated as,

$$\left| \mathbf{r}_R - \mathbf{r}_{P1} \right| = r \tag{27}$$

Table 1. Numerical parameters.

mass [kg]	M	18×10^3
inertia moment [kg/m^2]	I	1.7×10^4
effort arm[m]	R	1.4
radius of base [m]	r	1.3×10^{-2}
shape angle[rad]	φ	0.5
distance between G and P_0 [m]	x_0	0.7
distance between G and P_0 [m]	y_0	1.2
coefficient of dynamic friction	μ	0.3
coefficient of static friction	μ_0	0.7

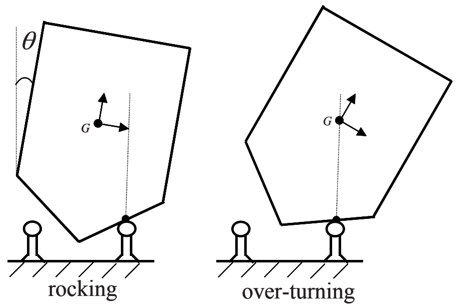


Fig. 3. Definition of rocking and overturning.

4. Numerical simulation

This section presents some numerical examples that employ the equations of motion described in the preceding section. The base is excited sinusoidally. Here, rocking motion both with and without slide is discussed. The numerical integration used in calculation is the Runge-Kutta method, with the integration timestep 10^{-6}s. The numerical parameters are shown in Table 1.

4.1 Influence of disturbance

The influence due to the frequency and the acceleration amplitude of the excitation on the rocking motion and the critical value for the occurrence of rocking or overturning are investigated here. In Figure.3, the occurrence of rocking is defined as the state in which the angular displacement becomes 0.01rad or above. On the other hand, the occurrence of overturning is defined as the state in which the projected center of mass of the rigid body crosses over the contact point. Here, the region of overturning is for the case of a simple rigid body model. Note that the condition for overturning of an actual railway system is not discussed in this paper. If the rigid body is overturned, the numerical simulation is terminated.

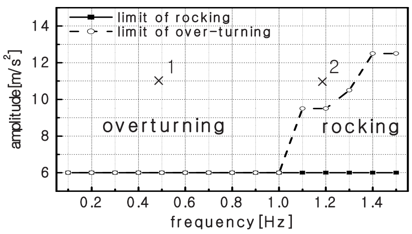
The acceleration amplitude during the occurrence of rocking and overturning is investigated under the excitation frequency from 0.1[Hz] to 1.5[Hz]. Fig. 4 show the cases with and without slide. Here the occurrence of rocking or overturning is judged in 10.0s. The coefficient of friction is 0.7.

4.1.1 Limit of rocking and overturning

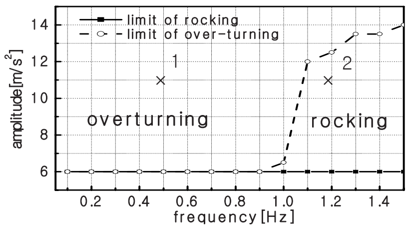
The trend of the Fig. 4 is discussed here. The region over the solid line represents rocking. Moreover, the region over the dotted-line represents overturning. In the region below both lines, the rigid body is in continual contact with both sides of the base. In the case of a low frequency, the rigid body immediately reaches the point of overturning. However, as the excitation frequency is gradually increased, the region in which rocking occurs becomes apparent. Thus, the acceleration amplitude for overturning increases rapidly at a high frequency. When the excitation frequency increases with a constant acceleration amplitude, the velocity amplitude decreases. In other words, the energy transferred to the rigid body also decreases.

4.1.2 Influence of slide on rocking and overturning

In Fig. 4, it is confirmed that it is easier to overturn in the case without slide than in the case with slide. Fig. 5 explains this mechanism from the viewpoint of the rotational moment around the center of mass. Here the acceleration amplitude is $11.0 m/s^2$ and the excitation frequency is 1.2Hz. Without slide, the constraint force acts on the contact point. With slide, the frictional force acts on the contact point. The constraint force is larger than the frictional force during rocking. In Fig. 5, it is clarified that the rotational moment due to the constraint force without slide is larger than that due to the frictional force with slide. To investigate the influence of slide from the time response, the motion of the rigid body at a high frequency is compared with the motion at a low frequency. These two cases are marked as 1 and 2 in Fig. 4(a) and (b), respectively. The acceleration amplitude of the excitation is $11.0 m/s^2$. The frequency of the excitation is given as 0.5Hz or 1.2Hz. The dotted lines in Figs. 6, 8, and 9 reveal the acceleration of the excitation. First, in the case with the low frequency both cases with and without slide are studied under an excitation frequency of 0.5Hz. The angular displacement is shown in Fig. 6. In the case of the frequency 0.5Hz, a slide is not generated even in the case where



(a) Without slide



(b) With slide

Fig. 4. Limit of rocking and overturning.

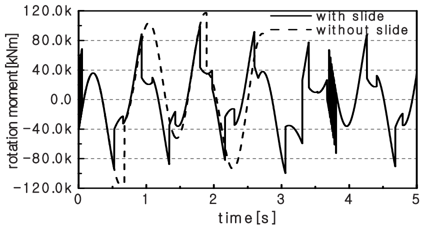


Fig. 5. Rotational moment around center of mass.

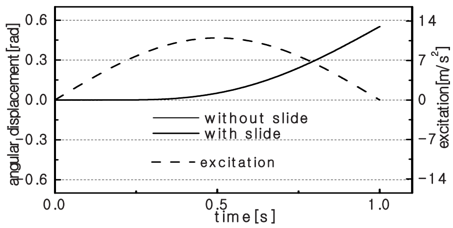


Fig. 6. Angular displacement of rigid body.

slide is taken into consideration. The transfer of the contact point is very small as shown in Fig. 7. The angular displacement then immediately becomes large, and the rigid body overturns around t=1.0s. Next, in the case of the high frequency 1.2Hz, the angular displacement without slide is shown in Fig. 8 and the case with slide is shown in Fig. 9. The transfer of the contact point with slide is also shown in Fig. 10. It is confirmed that the rigid body overturns in the case shown in Fig. 8, while rocking is maintained in the case shown in Fig. 9.

The reason for the overturning in the case without

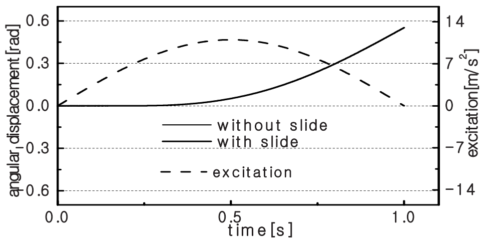


Fig. 7. Transfer of contact point.

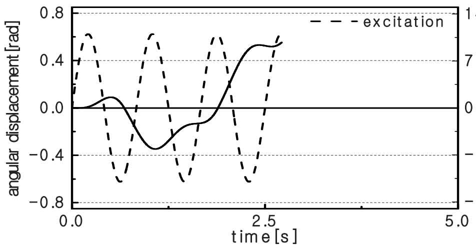


Fig. 8. Angular displacement of rigid body without slide.

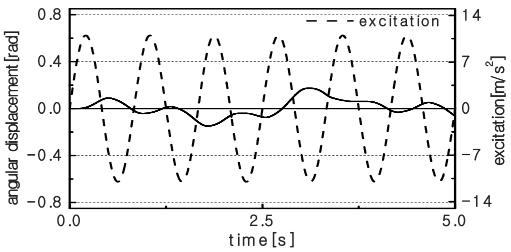


Fig. 9. Angular displacement of rigid body with slide.

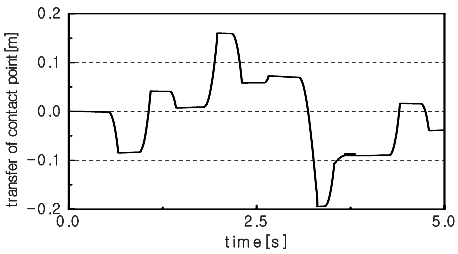


Fig. 10. Transfer of contact point with slide.

slide is that the excitation and the angular displacement of the rigid body are out of phase. So the angular displacement increases rapidly, and the projected center of mass crosses over the contact point. On the other hand, it can be seen in Fig. 9 that the motion of the rigid body is not always out of phase with the excitation. From 0s to 0.5s shown in Fig. 8 and 9, the angular displacement in the both cases with and without slide is almost the same. After 0.5s, the contact point slides markedly as shown in Fig. 10, and around the same time in Fig. 9, the angular displacement is suppressed by the slide. Therefore, rocking with slide maintains for a long time.

5. Conclusions

Planar rocking motion of a rigid body on a base with slide at the contact point is discussed as a unilateral contact problem. In this paper, the rigid body has a line at the contact point with a base whose shape is an arc. The motion of the rigid body with and without slide during rocking is investigated numerically.

Considering the influence of the frequency and the acceleration amplitude of the excitation on rocking motion, the critical values for the occurrence of rocking and overturning are investigated. It is qualitatively discussed that the occurrence conditions of rocking and overturning are different for low and high frequency of the excitations.

Also, the influence of slide on the occurrence of rocking and overturning is clarified, calculating the slide transfer of the contact point and the moment acting on the rigid body and the phase relation between the rigid body motion and the excitation.

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